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SOUND ATTENUATION IN LINED DUCTS

SIMPLIFIED DESIGN PROCEDURES

August, 1981

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ONTARIO MINISTRY OF THE ENVIRONMENT
NOISE POLLUTION CONTROL SECTION

ERRATA

"Sound Attenuation in Lined Ducts, Simplified Design Procedures", August 1981, J. Buur, C. A. Krajewski.

1. Line 14 from the bottom on page 1 (just under Figure 1.1) should read:

"The attenuation D in dB "

2. Equations (6), (7), (8) and (9) on page 6 are expressed in C.G.S. units. M.K.S. form of the equations is given on page 257 of the Reference 15.

3. Lines 3, 4 and 5 on page 9 should read:

"The resulting constriction of the flow causes an increase in acoustic impedance."

4. The paragraph under Figure 2.3. on page 9 should read:

"In such case, according to Ingard⁶, the resistivity is increased by $\frac{R_f \delta}{p}$, where the "end correction"

$\delta = R_f \cdot s \cdot f(p)$ is found according to the open area ratio p of the perforation, using Figure 2.4."

The next sentence, as well as the formula shown beside Fig. 2.4 should be deleted.

5. In equation (13) on page 10 the term r should be replaced by R_f .
6. Term D_h in equation (23) and in line 15 on page 13 should be replaced by D .
7. In Figures 3.3, 3.4. and 3.5. the input parameter $R=1,48 \times 10^5$ mks rayls/m should be replaced by equation (33) from page 17.
8. Line 4 on page 23 should read:
"c sound velocity (m/s) "
9. Line 6 on page 23 should read:
" D_h attenuation per duct length (dB) "
equal the duct height.
10. Line 9 on page 23 should read:
"k wave number = $\frac{\omega}{c}$ (radians/m) "

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SOUND ATTENUATION IN LINED DUCTS
SIMPLIFIED DESIGN PROCEDURES

Project Supervisor: C. A. Krajewski, P.Eng.
Draft Prepared by: Jacob Buur

ONTARIO MINISTRY OF THE ENVIRONMENT
POLLUTION CONTROL BRANCH
NOISE POLLUTION CONTROL SECTION

August 1981

SUMMARY

This is an attempt to review methods for the determination of acoustic impedance of various absorbent lining designs and the available procedures for calculating duct attenuation. A comparison of the available methods and the suggested design procedures are included in the body of the report.

The draft report is the work of Jacob Burr, a foreign student employed by the Ontario Ministry of the Environment, Toronto, under the International Association for the Exchange of Students for Technical Experience (IASTE) exchange program. At the time of his short 1981 (six weeks) work term with the Noise Pollution Control Section he had completed three years of training in electronic engineering at the Technical University of Denmark in Copenhagen.

The project was assigned, and work progress was supervised by C. A. Krajewski, P.Eng., Senior Acoustical Engineering Specialist in the Noise Pollution Control Section, Pollution Control Branch.

	<u>TABLE OF CONTENTS</u>	PAGE
1.	Introduction	1
2.	Determination of impedance for the duct lining	2
2.1.	Acoustic impedance	2
2.2.	Lining Design	3
2.3.	Parameters describing acoustic behaviour of absorptive materials	4
2.4.	Impedance of absorbing lining	5
2.5.	Thick layer of absorbent material	7
2.6.	Thin layer of absorbent material backed by air cavity	7
2.7.	Perforated facing	9
2.8.	Other types of lining	11
3.	Calculation of duct attenuation	12
3.1.	Morse and Cremer method	12
3.2.	Beranek method	14
3.3.	Kurze method	15
3.4.	Mechel method	16
4.	Comparison of methods	17
5.	Cylindrical ducts	21
6.	Optimal wall impedance	21
7.	Suggested design procedure	22
8.	List of variables	23
9.	References	25
	APPENDIX	27

1. INTRODUCTION

The problem of noise propagated through a duct which allows the passage of air or other fluids is usually associated with air handling or combustion equipment, such as; boilers, fans, combustion engines, etc. An effective method of reducing such noise is to line one or more of the inner walls of the duct with sound absorbing material, see Figure 1.1.

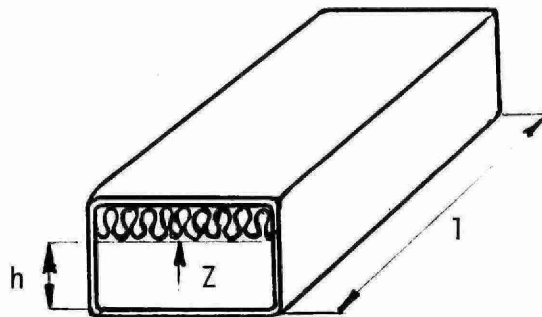


Figure 1.1.

The attenuation D in db afforded by such duct is a rather complex function of the duct geometry (i.e. the free height h and length l), the frequency of the sound, the temperature of the flowing medium, and the acoustic impedance of the absorptive lining material.

A number of approximations to the exact theory for calculating this attenuation exists, since the attenuation equation can only be solved analytically for simple cases.

An analysis of acoustic effectiveness for the lined duct involves two uncorrelated steps:

- Determination of the acoustic impedance Z for the absorbing wall(s).
- Calculation of duct attenuation based on its geometry and known value of the acoustic impedance.

As no comprehensive summary of design procedures for various types of ducts can be found in the available literature, an attempt was made in this paper to:

- (i) review theory related to determination of the impedance for various absorbent lining designs;
- (ii) review basic theory of sound attenuation in ducts;
- (iii) compare various approximate calculation methods;
- (iv) provide machine format for a practical design assessment procedure.

This project is limited to theoretical analysis as no duct equipment or measurement facilities are available for testing. It is suggested that compilation of experimental results from various sources on attenuation for lined ducts be an extension of the present study.

2. DETERMINATION OF IMPEDANCE FOR THE DUCT LINING

2.1. Acoustic Impedance

The specific acoustic impedance Z of an absorbing wall is defined as the ratio of sound pressure p to the particle velocity u at the same point of the medium.

$$Z = \frac{p}{u} \quad (\text{kg/sm}^2) \quad \text{or} \quad (\text{mks rayls}) \quad (1)$$

$$Z = R + jX$$

The impedance is a complex variable and consists of the resistance $R = \text{Re } Z$ and the reactance $X = \text{Im } Z$. The acoustic impedance of air Z_0 , often called the characteristic impedance of air, is:

$$Z_0 = \rho_0 \cdot c = 1.20 \text{ kg/m}^3 \times 343 \text{ m/s} = 413 \text{ kg/m}^2 \text{ s} \\ (\text{at } t = 20^\circ \text{C})$$

It is convenient at times to express the impedance as the normalized impedance Z_n :

$$Z_n = \frac{Z}{Z_0} = \frac{Z}{\rho_0 c} \quad (2)$$

2.2. Lining Design

The design of the lining is an important factor in achieving desired duct attenuation. Type of lining is selected to provide the optimum impedance at the frequency where greatest attenuation is required. However, other parameters such as temperature, pressure drop and dust load of the passing media must also be considered in the selection process. Figure 2.1. shows various commonly used lining designs.

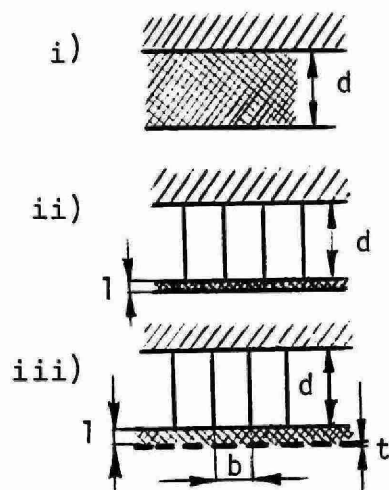


Figure 2.1.

For a homogeneous bulk of absorptive material in front of a rigid wall (see configuration i), the resistance is determined solely by the material, and the reactance is negative in all practical cases, as the air in the layer acts very much like a spring because of the rigid wall.

In the case of a thin layer of absorptive material at a distance d from the wall (see configuration ii), the air cavity adds to the imaginary part of the impedance. To avoid propagation of sound behind the absorbing layer, the airspace is often subdivided into cells by partitions.

By applying a thin sheet of perforated (hard) facing to absorptive material (see configuration iii), both the resistivity and the resistance changes. Most important, the mass of air in the holes of the perforated facing results in a positive contribution to the reactance, thus opening the possibility of tuning the system to a required resonance frequency.

2.3 Parameters Describing Acoustic Behaviour of Absorptive Materials.

Porous material used for the purpose of attenuating sound is normally composed of fibers (mineral wool, glass) or foam. To describe the acoustic behaviour of absorptive materials the following parameters are used:

- a) Flow Resistivity R_f defined as the ratio of sound pressure change through the material to the particle velocity u times thickness l .

$$R_f = \frac{\Delta P}{u \cdot l} \quad (\text{kg/sm}^3) \quad \text{or} \quad (\text{mks rays/m})$$

- b) Porosity P defined as the ratio of the volume of voids V_a to the total volume V_m .

$$P = \frac{V_a}{V_m}$$

In the case of fibrous material ¹⁵ the porosity is expressed as:

$$P = 1 - \frac{\rho_m}{\rho_t}$$

where ρ_m is the density of the material and ρ_t is the density of the fibres.

- c) Structure Factor m accounts for the increase in inertia of the gas in the material due to the inner structure of the bulk. There seems to be a way of measuring m, but Beranek ¹⁵ has given an approximate relation between structure factor and porosity of a homogenous material. See Figure 2.2.

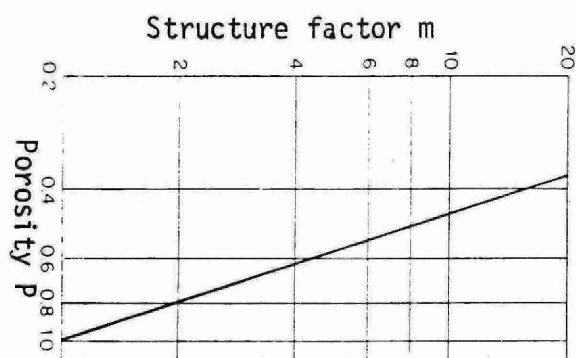


Figure 2.2.

2.4 Impedance of Absorbing Lining.

The acoustic impedance of an absorbing lining may be measured or approached theoretically.

Kurze ⁹ gives a method for measuring the complex impedance, and Dawern ¹² presents an impressive collection of data measured on a duct silencer system consisting of a blanket of porous material with perforated facing, backed by an airfilled cavity, (Figure 2.1.c.)

A theoretical analysis of acoustic impedance for dissipative lining materials has been conducted by Bolt, 1947², Ingard and Bolt 1951³, Callaway and Ramer 1952⁴, Ingard 1954⁶, Kurze 1965⁷, and Velizhanina 1965⁸. It generally involves the solution of the wave equation for a layer of absorbing material of thickness d backed by a rigid wall:

$$Z = \frac{Z_0}{\tanh yd} \quad (3)$$

where Z_0 is the characteristic impedance of the material and γ is the propagation constant of the material.

These coefficients are given explicitly by Velizhanina⁸, using flow resistance r , porosity P and structure factor m as defined earlier:

$$Z_0 = \sqrt{\frac{K_0}{P \cdot q} \left(\frac{m}{P} \rho - j \frac{r}{\omega} \right)} \quad (4)$$

and

$$\gamma = j \omega \sqrt{\frac{P \cdot q}{K_0} \left(\frac{m}{P} \rho - j \frac{r}{\omega} \right)} \quad (5)$$

where q is the specific heat ratio at constant pressure and constant volume, and K_0 is the compressibility of air.

For isothermal conditions in the material $q=1.4$ and for adiabatic condition $q=1$.

At low frequencies the conditions are likely to be isothermal. Through a number of measurements on a wide range of absorptive materials Delany and Bazley¹⁰ have empirically determined power relations for the characteristic impedance and the propagation coefficient which are both functions of the flow resistivity R_f :

$$Z_0 = R + jX, \quad \gamma = \alpha + j\beta$$

$$R = \rho c + 9.08 \rho c \left(\frac{f}{R_f} \right)^{-0.75} \quad (6)$$

$$X = -11.9 \rho c \left(\frac{f}{R_f} \right)^{-0.73} \quad (7)$$

$$\alpha = 10.3 \frac{\omega}{c} \left(\frac{f}{R_f} \right)^{-0.59} \quad (8)$$

$$\beta = \frac{\omega}{c} + 10.8 \frac{\omega}{c} \left(\frac{f}{R_f} \right)^{-0.70} \quad (9)$$

Thus, there are two different approaches to the determination of value Z for an absorptive lining, using either the analytically or the empirically derived functions for Z_0 and γ .

2.5 Thick Layer of Absorbent Material.

In the case of a thick layer of porous material backed by a rigid wall, as shown in Figure 2.1.a., equation (3) may be applied directly.

2.6 Thin Layer of Absorbent Material Backed by Air-cavity.

This type of wall design, shown in Figure 2.1.b. has, accordingly to Velizhanina⁸, the acoustic impedance:

$$Z = \frac{r}{3 P_q} (P_q l + d) + j \left[\omega \frac{\rho m}{3 P_q^2} (P_q l + d) - \frac{\rho c^2}{\omega (P_q l + d)} \right] \quad (10)$$

The formula is the result of expanding the $\tanh$ in a series and retaining only the two first terms. The last part of the reactance represents the stiffness of the air in the cavity between material and wall, similar to the formula for a Helmholtz resonator, $(P_q l + d)$ being the depth of the resonator. The first part of the reactance is the mass of air $(\rho \cdot l)$ moving in the layer of material. The "torturous path" the air has to make through the layer is taken into account by multiplying the first component of the imaginary part by the structure factor m . At low frequencies this component will normally be small compared to the stiffness of air.

Ingard and Bolt³ give a simpler approximation of the impedance of the same wall configuration, as follows:

$$Z = r_l + j \left[\omega \rho m_l - \frac{\rho c}{\tan\left(\frac{\omega}{c} d\right)} \right] \quad (11)$$

This equation is derived on the assumption that the porous layer is very thin compared to the wavelength of sound, i.e. $kl \ll 1$. In such case the resistivity of the material is simply r_l . It is recognized that the stiffness of air in the cavity is here expressed by the equation used for the one-dimensional case of a piston radiating into a narrow tube

$$\frac{\rho c}{\tan\left(\frac{\omega}{c} d\right)}$$

This holds only if the partitions dividing the cavity into cells are close enough together compared to the wavelength, so that wave motion may be assumed in one direction only. This is likely to be the case when spacing $b < \lambda/4$. If b is larger or no partitions are present, the term becomes:

$$\frac{\rho c}{\tan\left(\frac{\omega}{c} d \cos\theta\right) \cos\theta} \quad (12)$$

where θ is the angle of incidence of plane wave (normal incidence: $\theta = 0^\circ$ and $\cos\theta = 1$).

This expression may only be used at low frequencies as shown by comparing the expressions below (See Appendix A.1.)

$$\frac{c}{\omega \cdot d} \quad \text{and} \quad \frac{1}{\tan\frac{\omega}{c} \cdot d}$$

2.7 Perforated Facing

Perforated facing applied to an absorbing wall distorts the flow in a small region around the perforation. The resulting constriction of the flow causes an increase in acoustic mass reactance.

If the facing is in close contact with the porous material i.e. the airspace between facing and material

δ_t is smaller than the diameter $2a$ of the holes (see Figure 2.3.ii) and iii) the effect of the facing is not only mass reactance but a resistance as well.

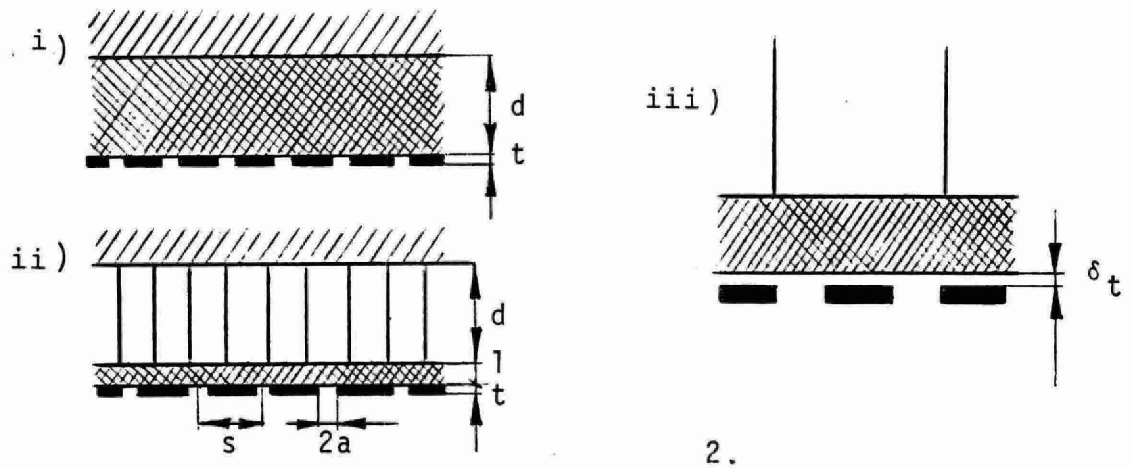
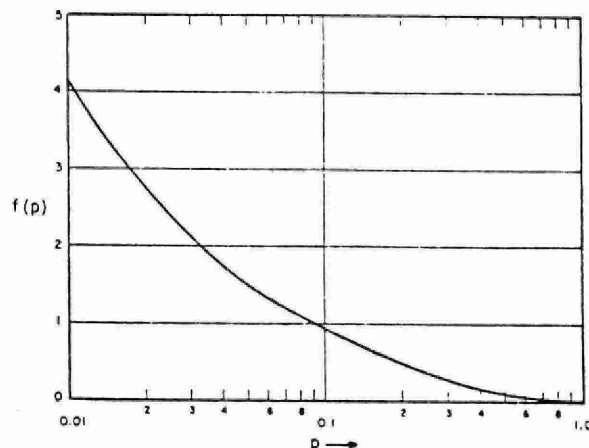


Figure 2.3.

In such case, according to Ingard⁶, the resistivity is increased by $\frac{r\delta}{p}$ where the "end correction" δ is found from Figure 2.4. according to the percentage open area of the perforation p . The resistivity may be even larger than the resistivity due to the material.



$$\delta = s \cdot f(p)$$

where s is the distance between holes

Figure 2.4.

The mass of air in the holes results in the reactance $(t + (1+m) \delta) / p$ to be added to the impedance of the wall without facing Z_b , as expressed by equations (3), (10) and (11):

$$Z = Z_b + r \delta / p' + j \omega \rho (t + (1+m) \delta) / p' \quad (13)$$

If the facing is not in close contact with the absorbent layer, the resistance contribution to the impedance is negligible, but reactance remains the same.

Velizhanina⁸ uses a slightly different "end correction" δ_v given by:

$$\delta_v = 0.85 a (1 - 2.50 a/s) \quad (14)$$

Thus, the impedance is expressed by:

$$Z = Z_v + r \delta_v / p' + j \omega \rho (m + 1) \delta_v / p' \quad (15)$$

where Z_v is found in equation (10)

A viscous resistance from the holes is also present, although it is negligible in most practical cases. According to Bolt² the value of viscous resistance is approximately:

$$R_v = 4 t / a \sqrt{\pi \rho q_b f} \quad (16)$$

where q_b is the coefficient of viscosity of air.

2.8 Other Types of Lining.

Kurze⁷ gives two other examples of absorbent walls:

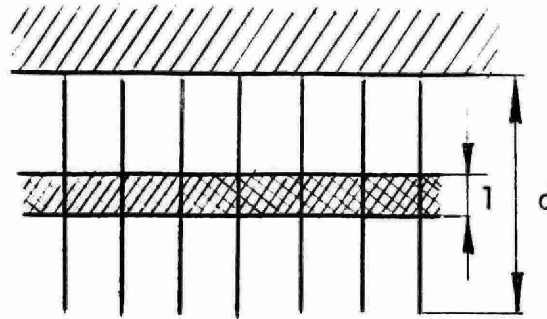


Figure 2.5.

- (i) The thin layer of absorbent material is located halfway between the wall and the end of partitions as shown in Figure 2.5. For this type of lining the impedance is given by:

$$Z = \frac{1 - \cot^2 kd/2 - jr l \cot kd/2}{rl - j 2 \cot kd/2} \rho c \quad (17)$$

- (ii) A set of Helmholtz resonators with resonant frequency ω_{res} , as shown in Figure 2.6.

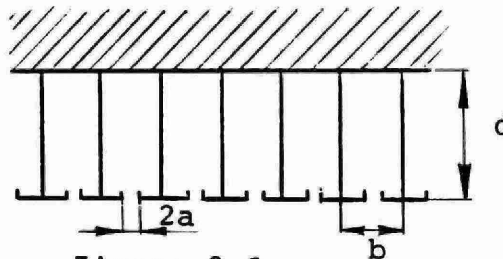


Figure 2.6.

For this type of lining the impedance is given by:

$$Z = R \cdot b^2 + j \frac{\rho c^2}{\omega_{res} d} \left(\frac{\omega}{\omega_{res}} - \frac{\omega_{res.}}{\omega} \right) \quad (18)$$

where

$$R = \frac{1}{\pi a^2} \rho \sqrt{2 \omega \mu} \left[\frac{t}{a} + 2 \left(1 - \frac{\pi a^2}{b^2} \right) \right]$$

3. CALCULATION OF DUCT ATTENUATION

The theory of the transmission of sound inside ducts has been studied by Morse, 1939¹, Cremer, 1953⁵ and Kurze, 1965⁷ and 1969⁹. All three considered the case of a duct of rectangular cross-section with one absorbing wall and the other three walls perfectly rigid. The derived formulas also apply to the case of two equally lined walls facing each other on basis of the mirror image principle, provided the dimension h , the free duct height, is defined as in Figure 3.1.

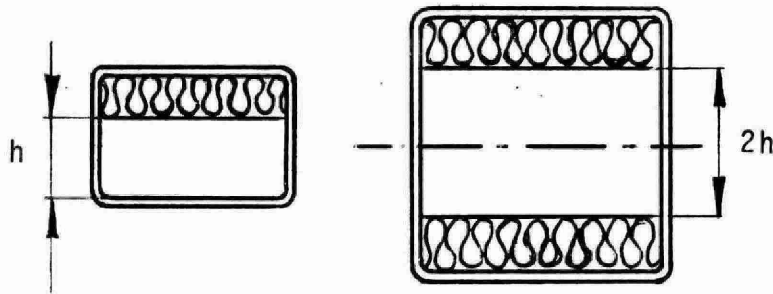


Figure 3.1

The attenuation of treated duct is usually expressed as D_h in dB per unit length equal to free duct height h .

3.1 Morse and Cremer Method

Morse solves the wave equation for two dimensional case (x, y coordinates) and the simplest type of wave:

$$p = P_0 \cosh \left[(\pi y/h)(\kappa - j\mu) \right] \exp \left[j\omega t - (\omega/c)(\sigma + j\tau)x \right] \quad (19)$$

defining the transmission parameter $(\sigma + j\tau)$ and the distribution parameter $(\mu + j\kappa)$ as follows:

$$(\sigma + j\tau)^2 = \left[(\mu/\eta) + j(\kappa/\eta) \right]^2 - 1 \quad (20)$$

$$\eta(\sigma + j\tau) = \sqrt{(\mu + j\kappa)^2 - \eta^2}$$

where η is the frequency parameter;

$$\eta = \frac{2hf}{c} \quad (21)$$

Applying a set of boundary conditions to eq. (19), Morse determines the relationship between the distribution parameter $(\mu + j\kappa)$ and the acoustic impedance Z of the absorbing wall:

$$\frac{Z}{\rho c \eta} = \frac{\cot h[-j\pi(\mu + j\kappa)]}{\mu + j\kappa} \quad (22)$$

The attenuation in dB/m is determined by:

$$D_h = 8.686\pi\eta\sigma = 8.686\pi \operatorname{Re} \{ \sqrt{(\mu + j\kappa)^2 - \eta^2} \} \quad (23)$$

Since it is not possible to find $(\mu + j\kappa)$ analytically from equation (22), Morse introduced nomograms which show the distribution parameters for a given impedance $|Z|$, phase angle ϕ and frequency parameter η .

The nomograms, however, are not suitable for design purposes, since D_h still has to be found by applying equation (20). Therefore, Cremer⁵ presented graphs which yield the value of σ for a set of values for η , $\frac{Z}{\rho c \eta}$ and ϕ .

Furthermore, Cremer shows that types of waves of higher order than the one considered by Morse will be attenuated more effectively than calculated. On the other hand, high frequency waves will tend to form "rays" close to the center of the tube. This effect causes the attenuation D to decrease proportional to η^{-1} and even η^{-2} . This effect should be considered if

$$\frac{|Z|}{\rho c \eta} (n + 1/2) < 0.3 \quad (24)$$

where n is the mode of the sound wave.

Further, Cremer states that while

$$\frac{\rho c}{Z} \eta < \frac{1}{8} \quad (25)$$

the problem may be solved as a one-dimensional case and thus equation (22) is simplified to:

$$(\mu + j\kappa)^2 = j \frac{\eta}{\pi} \frac{\rho c}{Z} \quad (26)$$

3.2 Beranek Method

For the study of duct silencers, using various absorptive lining configurations, Kurze⁷ has slightly altered Cremer's⁵ graphs to show actual D_h values in dB's instead of the parameter σ . These graphs have been used by Beranek¹⁵ to develop a procedure for calculating the attenuation of a lined duct. The procedure is outlined in the following steps:

1. calculation of the impedance Z at η
2. calculation of modulus $|Z|$ and argument θ
3. determination of the appropriate value of D_h applying $\frac{|Z|}{\rho c \eta}$ and θ to the graph (see Figure 3.2).

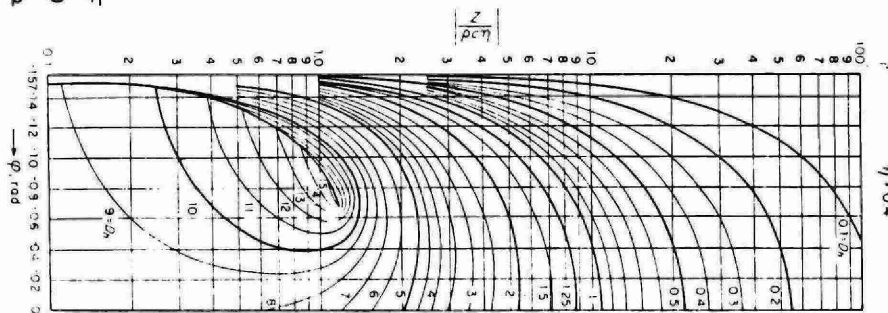


Figure 3.2
One of a number of graphs drawn by Kurze⁷ to obtain D_h as a function of the lining impedance.

This method represents a fair approximation to exact theory but it is rather cumbersome for practical applications. By computerizing the first two steps, D_h can be evaluated for a number of data sets in order to compare the methods discussed in the following two sections.

3.3 Kurze Method

Kurze derives a formula based on an assumption that the lining is of periodic structure. Considering the sound waves only at a number of discrete points and applying complicated matrix calculation the following equation is derived:

$$\sum_{v=0}^n D^v \left(r_v + s_v \frac{4nZ}{2nZ + jkh \rho c} \right) = 0 \quad (27)$$

$$\text{where } D^v = \left(\frac{kh}{n} \right)^2 - 2 + \left(\frac{\gamma h}{n} \right)^2$$

and r_v, s_v are the coefficients of the Tschebyscheff function associated with this equation, and n is the number of discrete points along the dimension h . Letting $n \rightarrow \infty$, the equation (27) approaches the equation derived by Morse¹ and Cremer⁵.

According to Kurze, the equation (27), for $n = 2$, yields results within 5% of those obtained using Beranek's graphical method. The complex propagation constant given by Kurze is shown in the following equation:

$$\gamma = jk \sqrt{1 - \left(\frac{2}{kh} \right)^2 \left[1 + \frac{1}{1 + \frac{4Z}{jkh\rho c}} \pm \sqrt{1 + \frac{1}{\left(1 + \frac{4Z}{jkh\rho c} \right)^2}} \right]} \quad (28)$$

The $+$ sign in the equation (28) is chosen so as to yield the smallest attenuation D_h , which is given by;

$$D_h = 8.686 \operatorname{Re}\{\gamma h\} \quad [\text{dB}]$$

Substituting $H = jkh\rho c \frac{1}{Z}$, duct attenuation is expressed by:

$$D_h = 8.686 \operatorname{Re}\{\sqrt{E - (kh)^2}\} \quad (29)$$

$$\text{where } E = \frac{16 + 8H \pm \sqrt{256 + 128H + 32H^2}}{4 + H} \quad (30)$$

Equation (30) is in a suitable form for machine analysis.

3.4 Mechel Method

A formula similar to equation (29) but containing different numerical coefficients is given by Mechel ¹¹ as follows:

$$D_h = 8.686 \operatorname{Re}\{\sqrt{E - (kh)^2}\} \quad (31)$$

where

$$E = \frac{105 + 45H \pm \sqrt{11025 + 5250H + 1650H^2}}{20 + 2H} \quad (32)$$

Dividing E by 6.5, it becomes clear that Mechel's coefficients are close to those shown in equation (30) except for denominator $20 + 2H$;

$$E = \frac{16.2 + 6.9H \pm \sqrt{260.9 + 124.3H + 39.1H^2}}{3.1 + 0.3H}$$

This indicates that the formula is derived in a way similar to Kurze's. Unfortunately, details of Mechel's theoretical derivations were not published and it is, therefore, difficult to determine limitations of his method. According to Mechel, the calculated values are always within 5% of the exact solution.

4. COMPARISON OF METHODS

For the purpose of comparison among the above discussed methods, the following simplified impedance equation was used:

$$Z = 3 \rho c - j 46 \rho c / f d \quad (33)$$

i.e. the same as given by Beranek¹⁵.

Attenuation values, calculated by all three methods for the ratio d/h of 0.25, 1.0 and 4.0 are shown in Figures 3.3, 3.4, and 3.5. The results obtained using Kurze's formula and simplified Mechel's formula are almost identical in the low and medium frequency range. Slight discrepancies appear above the frequency of 1 KHz, with Kurze's formula predicting approximately 0.08 dB (per unit length equal to the height of the duct) lower values than Mechel's simplified formula.

Attenuation values obtained using Beranek's nomogram method are, in the low frequency region up to 500 Hz, generally lower than those predicted using the other two methods by 0.15 to 0.2 dB. This seems to be compensated by higher attenuation values in the frequency region above 500 Hz. The closest agreement among all three methods is noted for higher ratio of d/h , and above the frequency of 500 Hz.

FIGURE 3.3.

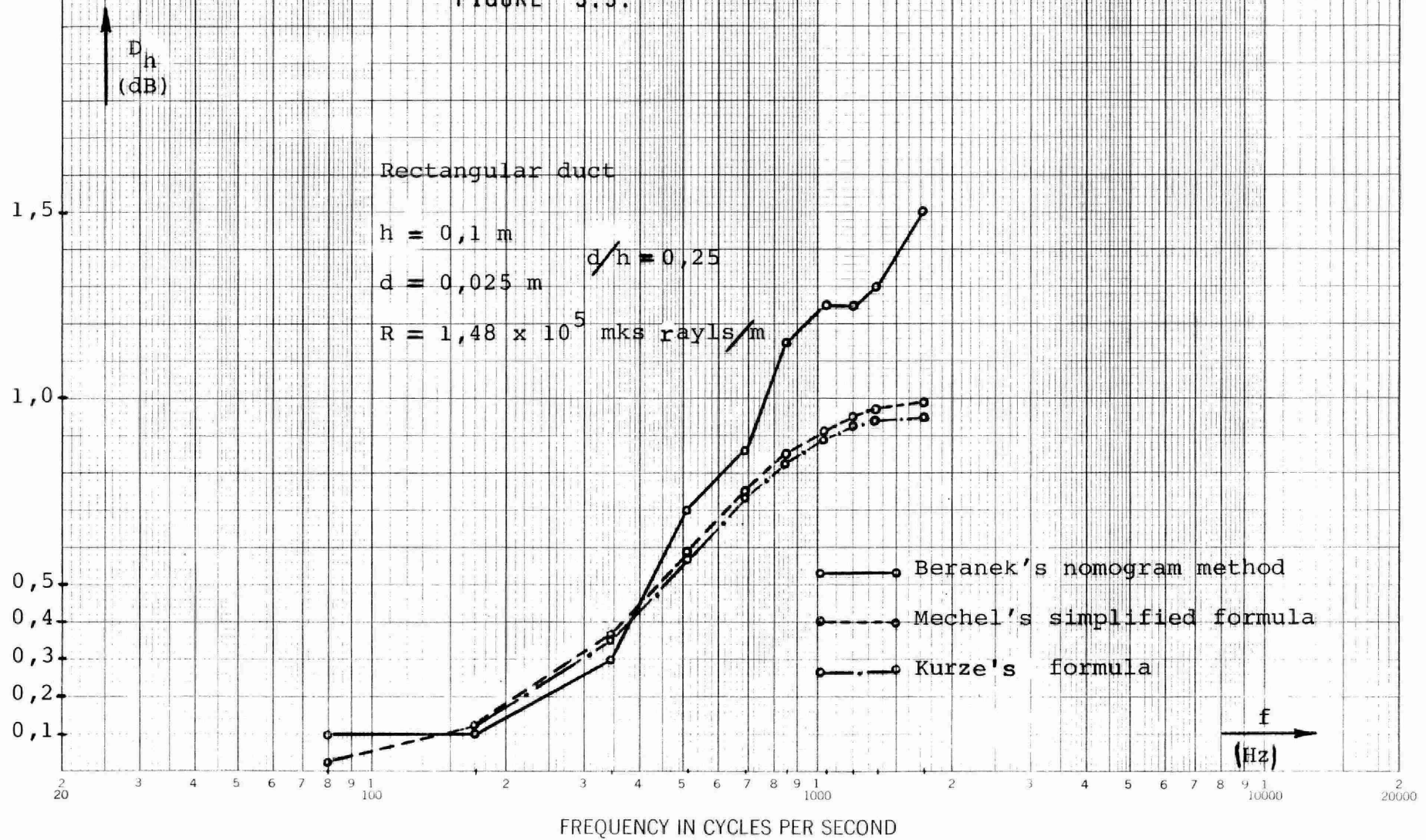


FIGURE 3.4.

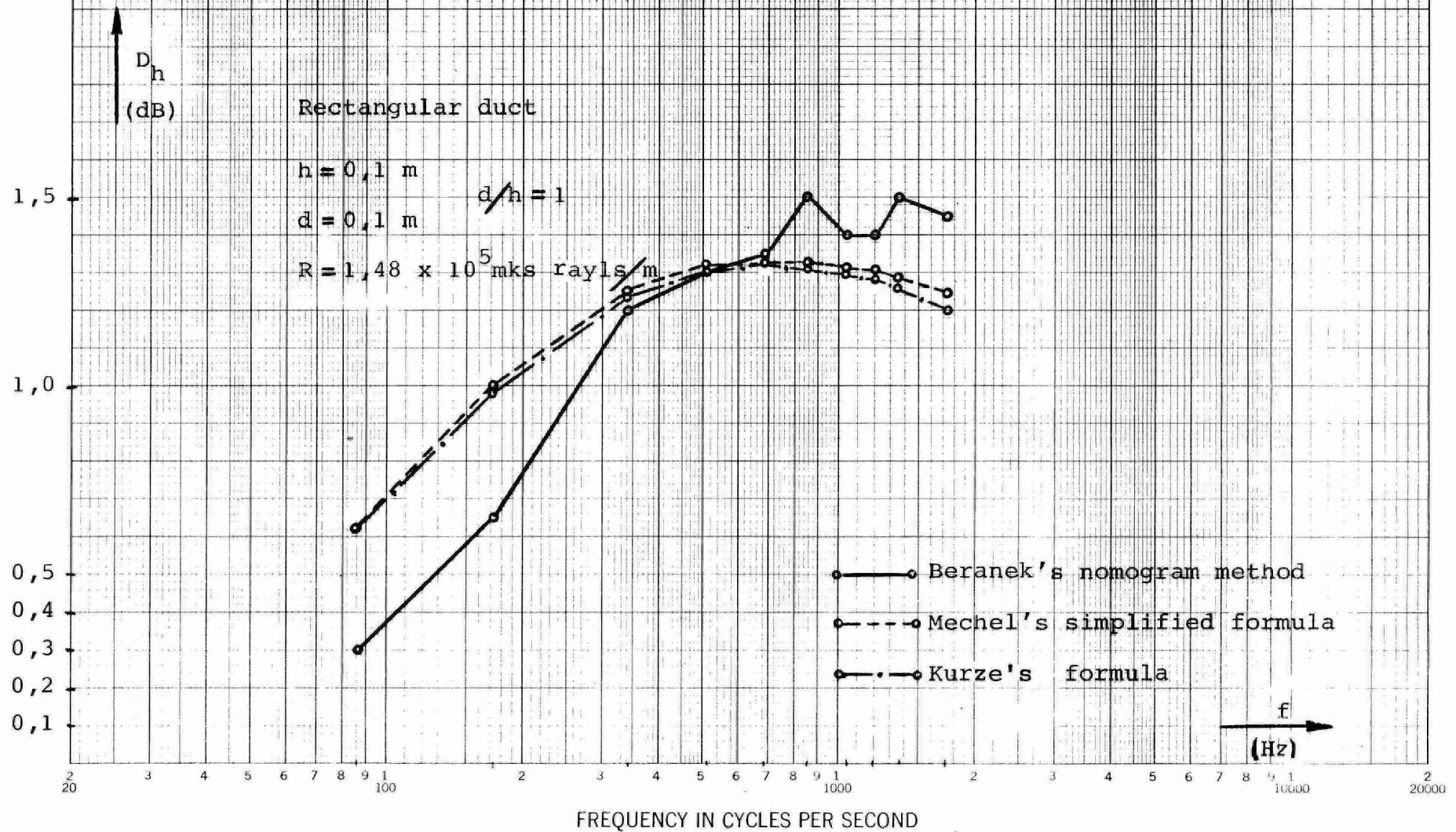
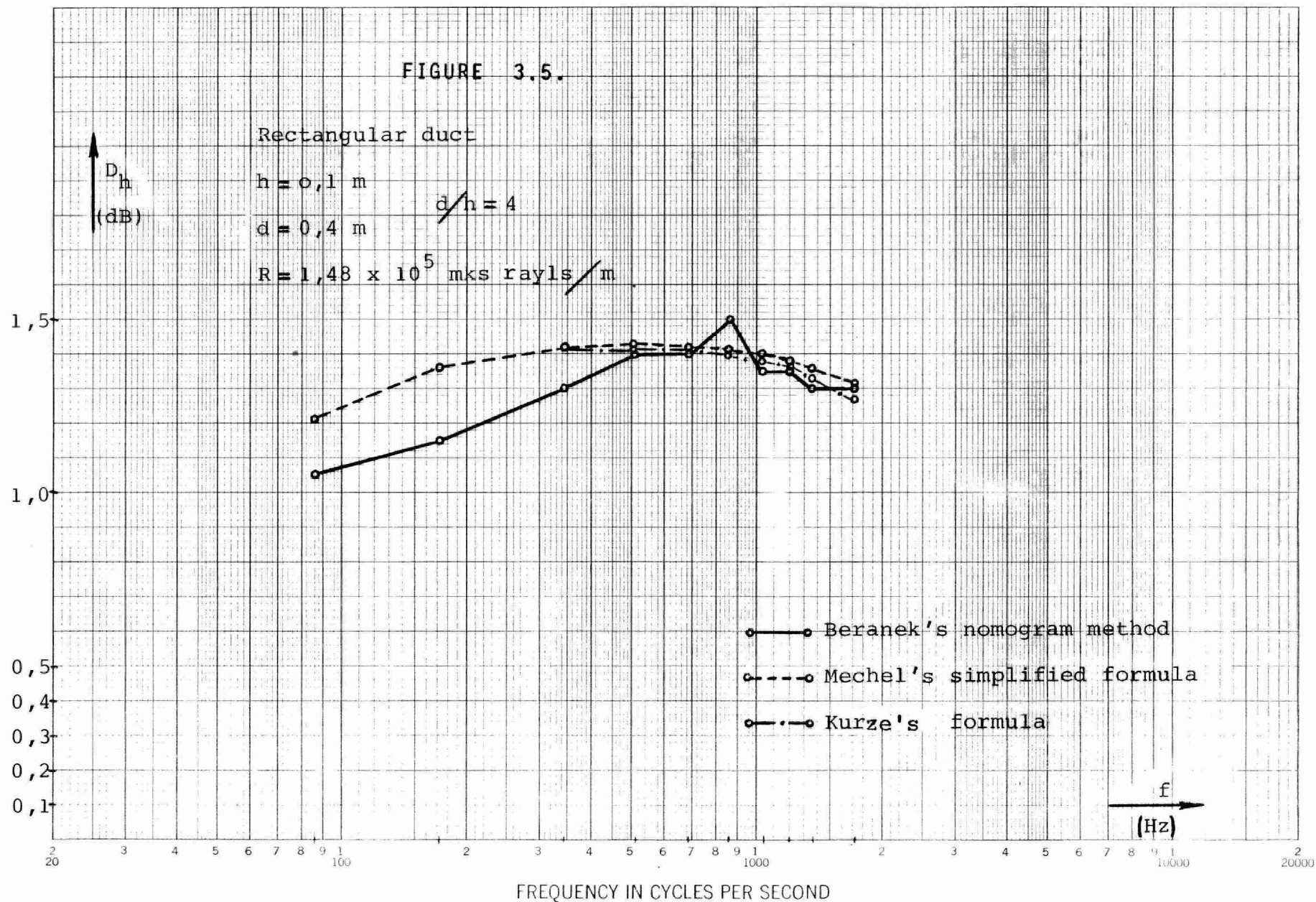


FIGURE 3.5.



5. CYLINDRICAL DUCTS

Morse¹ approaches the problem of ducts with circular cross-section and gives the solution in the form:

$$\frac{Z}{\rho c n} = \frac{A - jB}{-j(C - jD)(\mu + j\kappa)} \quad (34)$$

where $(\mu + j\kappa)$ is defined by equation (26) and A, B, C, and D are all series of Bessel functions.

Mechel¹¹ derives a formula quite similar to his own formulas for the rectangular duct. Only the quantity h was changed from duct height to radius, and the numerical coefficients are different. It is presented in the following form:

$$D_h = 8.686 \operatorname{Re} \{ \sqrt{E - (kh)^2} \} \quad (35)$$

where

$$E = \frac{96 + 36H \pm \sqrt{9216 + 2304H + 912H^2}}{12 + H}$$

A comparison between results of calculation (using the equation 35) and experimental results of measurements presented by Mechel, show rather good agreement.

6. OPTIMAL WALL IMPEDANCE

According to Cremer⁵, the impedance

$$Z = (0.91 - j 0.76) \rho c n \quad (36)$$

yields the highest attenuation.

Since this attenuation is rather difficult to achieve over the entire frequency range, the design of a duct silencer normally involves selection of a lining which will provide the optimum impedance at a given resonance frequency.

The highest attenuation values will be achieved with the lowest acceptable value of duct height h . Cremer⁵ also states that attenuation is greater the "closer the absorbent lining is to the sound propagation". A rectangular duct is, therefore, preferred to a round one.

7. SUGGESTED DESIGN PROCEDURE

7.1. Calculation of acoustic impedance for the selected duct design:

case of:

- (i) Homogeneous bulk of absorptive material in front of a rigid wall. (Figure 2.1.a),
Use equations (3), (6), (7), (8), and (9).
- (ii) Thin layer of absorptive material at a distance d from a rigid wall (Figure 2.1.b.),
Use equation (11)
- (iii) Thin sheet of perforated facing applied to absorptive material (Figure 2.1.c.),
Use equation (13)
- (iv) Absorbent wall (Figure 2.5.),
Use equation (17)
- (v) Set of Helmholtz resonators (Figure 2.6.),
Use equation (18)

7.2 Calculation of duct attenuation:

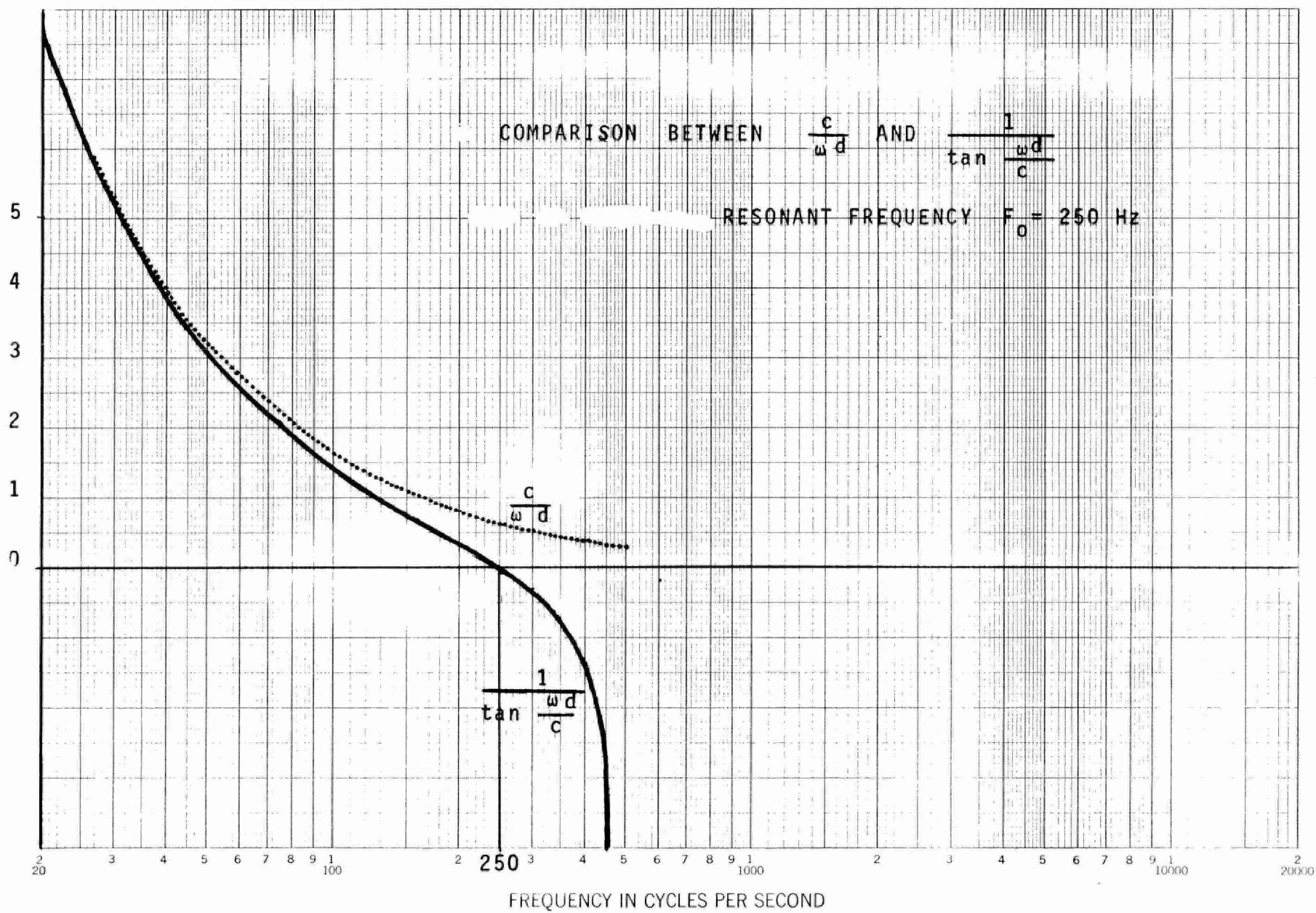
- (i) Rectangular ducts,
Use equations (28), (29), and (30).
- (ii) Cylindrical ducts,
Use equations (35) and (36)

<u>LIST OF VARIABLES</u>		<u>UNIT</u>
a	radius of holes in perforated facing	(m)
b	space between partitions	(m)
c	sound velocity	(m)
d	thickness of absorbing wall	(m)
D_h	attenuation per duct length = $D \frac{h}{l}$	(dB/m)
f	frequency	(s ⁻¹)
h	height of duct	(m)
k	wave number = $\frac{\omega}{c}$	(m)
K_o	compressibility of air	
l	thickness of absorbing blanket	(m)
m	structure factor	
p	sound pressure	(N/m ²)
R_f	flow resistivity	(mks rayls/m)
r	flow resistance	(mks rayls)
t_c	temperature	(°C)
u	particle velocity	(m/s)
X	acoustic reactance	(mks rayls)
z	normalized impedance = $\frac{Z}{\rho \cdot c}$	
Z	specific acoustic impedance	(kg/m ² s)
Z_o	characteristic impedance	(kg/m ² s)
n	frequency parameter = $\frac{2hf}{c}$	
γ	propagation constant	
$\mu + j\kappa$	distribution parameter	
ρ	density of air	(kg/m ³)
$\sigma + j\tau$	transmission parameter	
λ	wavelength	(m)
x	co-ordinate parallel to duct axis	

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